Grade 8 TN Lesson: Systems of Equations I

Britannica® Mathematics in Context

Use with MiC unit *Graphing Equations* after page 41

OR

Use with MiC unit *Algebra Rules!* after page 32

TN Standard: MA.8.SPI 0806.3.1

Find solutions to systems of two linear equations in two variables.

ENCYCLOPÆDIA Britannica®

Systems of Equations I

Sale!

Antonia and Jason are friends. Their favorite store, which sells only jeans and T-shirts, is having a gigantic sale. Antonia and Jason have saved some money, and they are ready to shop! For this sale, all T-shirts have one price and all jeans have one price.

Antonia buys two pairs of jeans and five T-shirts for $154.

1. a. Find a possible per-item price for jeans and for T-shirts.

b. Are other prices possible? Explain your answer.

c. Is it possible that the price of a T-shirt is $32? Explain why or why not.

2. a. What is the cost of four pairs of jeans and ten T-shirts?

   b. What are some other purchases for which you know the cost?

You can write a "shopping equation" for Antonia's purchase. If J stands for the jeans price and T for the T-shirt price, you can write:

$$2J + 5T = 154$$

This shopping equation is an example of an equation with two unknowns.

3. What shopping equation describes the price of four pairs of jeans and ten T-shirts? How is it related to the original shopping equation?

The equation $2J + 5T = 154$ is true for many values for J and T.

4. Check that the number pair $J = 52$ and $T = 10$ makes the equation true. Find three other number pairs that work.

5. Another number pair that works for the equation is $J = 82$ and $T = -2$.

a. Explain why these values do not make sense for this problem.

b. Describe the ranges of values for J and T that make sense for this equation.

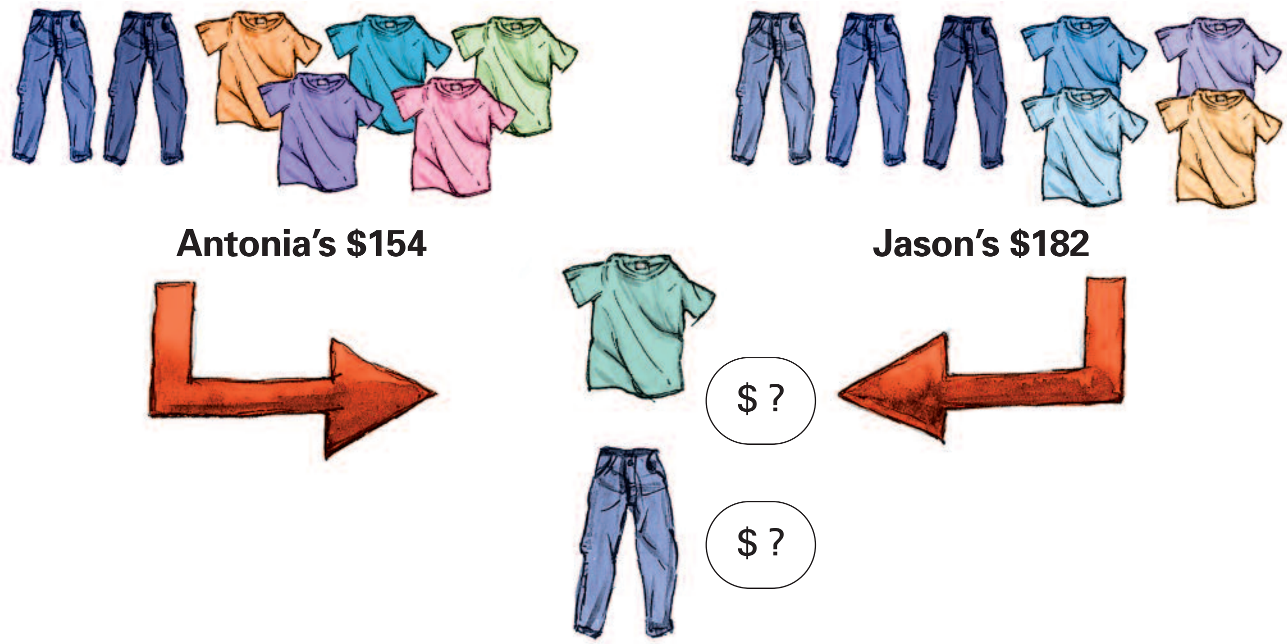
Values for J and T that make the equation true are called *solutions* to the equation.

6. Jason buys three pairs of jeans and four T-shirts. The total cost is $182.

a. Write a shopping equation for Jason's purchase.

b. Find three solutions without considering Antonia's purchase.

7. Look back at the information you have for Antonia's and Jason's purchases. Using that information, find the prices for one T-shirt and one pair of jeans.



Tables

	J	T	Price
Antonia's purchase	2	5	154
Jason's purchase	3	4	182
	6	15	462
	6	8	364
	0	7	98
	0	1	14
	0	5	70
	2	0	84
	1	0	42

In the *Comparing Quantities* unit, you developed several strategies to solve similar shopping problems. For one strategy, called "notebook notation," information is organized in a table.

Hannah solved problem **7** using the notebook notation method, as shown on the left.

8. Explain Hannah's solution.

9. Copy Hannah's table and write an equation for each row in the table.

Selena discovered her own way to solve problem **7**.

ANTONIA			JASON		
J	T	Price	J	T	Price
2	5	154	3	4	182
4	10	308	6	8	364
6	15	462	9	12	546
8	20	616	12	16	728
10	25	770	15	20	910

	J	T	Price
	6	15	462
subtract	6	8	364
divide by 7	0	7	98
	0	1	14

The value of T is 14.

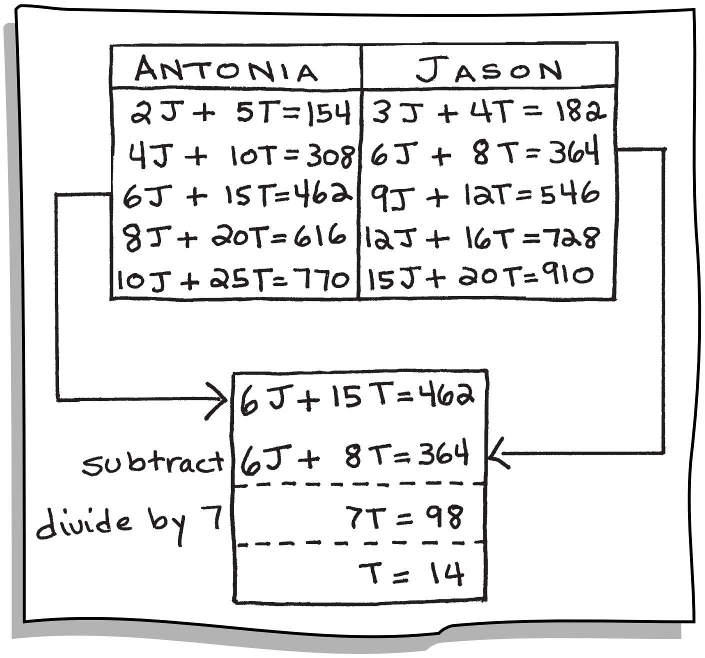
10. a. Explain Selena's strategy.

b. Describe how you can find the value of *J*.

11. Bard's Used Book Store is having a sale on paperback books and audio books (books on CD). All the books on sale are one price, and all the audio books on sale are one price. Amera buys two books and five audio books for \$46. Beth buys six books and three audio books for \$42. Find the price of one book and the price of one audio book.

12. The Jewelry Shop has one price for any watch and one price for any ring. Five watches and four rings cost \$134. Four watches and five rings cost \$127. Find the price of one watch and the price of one ring.

Below is another way to look at Selena's strategy from page 5. While it uses equations, it is still the same strategy.



13. **a.** You can find the value of J by combining the equations $8J + 20T = 616$ and $15J + 20T = 910$. Show how.

b. Once you know that $T = 14$, there is another way to find the value of J. Show how.

In problem **13**, you found a pair of values that satisfies both equations. The pair of values is called a *common solution* for the two equations.

14. **a.** Find a common solution for the two equations $X + 2Y = 95$ and $X + Y = 55$. Is there more than *one* common solution?

b. Do the equations $X + 2Y = 95$ and $3X + 6Y = 290$ have a common solution? Explain.

c. Is there a common solution for these three equations: $X + 2Y = 95$, $X + Y = 55$, and $3X + Y = 110$? Explain your answer.

d. Sandy thinks three equations can never have a common solution. Explain why you agree or disagree with Sandy.

15. A bill for two glasses of cranberry juice and three glasses of orange juice is \$5.20. Another bill for four glasses of cranberry juice and six glasses of orange juice is \$10.40.

a. Explain why it is not possible to find the price of one glass of cranberry juice.

b. Suppose the price of one glass of orange juice is 40 cents more than the price of one glass of cranberry juice. What is the price of one glass of cranberry juice?

Summary

In this section, you solved shopping problems. You used both a table (notebook notation) and equations to solve the problems.

The equation $2X + 5Y = 100$ has two unknowns, so it has many solutions. When a pair of values satisfies two equations with two unknowns, that pair of values is a *common solution*.

Check Your Work

Here are two equations with two unknowns:

$$2X + 5Y = 100$$

$$3X + 8Y = 156$$

16. This pair of equations has one common solution. Find it using a method discussed in this section.

For Further Reflection

Does every pair of equations with two unknowns have one common solution? Give examples to justify your answer.

Solutions and Samples

1. a. Many answers are possible. Sample responses:

A pair of jeans costs $27, and a T-shirt costs $20.

A pair of jeans costs $30, and a T-shirt costs $18.80.

A pair of jeans costs $22, and a T-shirt costs $22.

b. Yes. There are many possible price combinations that would allow two pairs of jeans and five T-shirts to cost $154.

c. No. If the price of a T-shirt is $32, five T-shirts would cost $160, which is already more than the total cost of $154 for two pairs of jeans and five T-shirts.

Hints and Comments

Overview

Students are introduced to the context of shopping. They find the prices of individual items when they know the total price of a combination of items.

About the Mathematics

The mathematics in this section is the same as in the unit *Comparing Quantities*. Students find possible prices of individual items based on the total price of a combination of two items. Some students may use a guess-and-check method to find one solution. Other students may think that they are purchasing seven items for $154. If all items sold for the same price, each would cost $22. Other students might continue with this line of thinking and adjust the $22 price so that the price of the jeans is higher.

Comments About the Solutions

1. Throughout this section, allow students to solve each problem using a strategy of their choice. It is more important that students be comfortable using one strategy to solve problems than it is for them to learn all of the strategies presented in this section. Exposure to different strategies should spark students' interest in an alternate solution strategy. Some students may find only a single solution to this problem. Some students may notice that if the price of jeans increases by $5, the price of a T-shirt decreases by $2. Have students compare their answers.

Extension

You may want to have students solve some of the problems from the unit *Comparing Quantities* that are similar to the problems on page 1 of this lesson.

Solutions and Samples

2. a. \$308. Strategies will vary. Students should note that the combination of four pairs of jeans and ten T-shirts is twice two pairs of jeans and five T-shirts. So, the new price equals $2 \times \$154 = \$308$.

b. Answers will vary. Sample responses:

Six pairs of jeans and 15 T-shirts cost \$462.

Eight pairs of jeans and 20 T-shirts cost \$616.

Ten pairs of jeans and 25 T-shirts cost \$770.

(Students can find these answers by adding two pairs of jeans and five T-shirts for \$154 to each successive purchase.)

3. $4J + 10T = 308$. This equation is twice the original equation.

Hints and Comments

Overview

Students use shopping equations to represent the combinations of jeans and T-shirts and their corresponding prices.

About the Mathematics

Students should be comfortable with equation notation, which was introduced and used in previous algebra units.

In the shopping equations, the letters stand for fixed, unknown prices and are called *unknowns*. Letters in algebra are often referred to as *variables*. Strictly speaking, the letters in the equations on page 2 of this lesson do not stand for variable values. Instead, the values are fixed but unknown.

Comments About the Solutions

2. a. While students are working on this page, you may want to check their understanding of equation notation. Make sure students understand how to read and write equations. It is important that students be able to explain the meanings of the numbers and letters in equations in their own words.

If students use their prices from problem **1a**, make sure they also notice the doubling relationship. A combination of twice as many of each item costs twice as much.

3. When the term *equation with two unknowns* is first introduced, ask students, *Why is it called an equation with two unknowns? What are the unknowns?* [It is called an *equation with two unknowns* because it contains two values that are unknown. The unknowns are J, the price of each pair of jeans, and T, the price of each T-shirt.]

Solutions and Samples

4. $(2 \times \$52) + (5 \times \$10) = \$104 + \$50 = \$154$

Students' number pairs will vary. One possible exchange that keeps the price the same is $5 less on the jeans for $2 more on the T-shirts. Sample answers:

Jeans	T-shirts
$47	$12
$42	$14
$37	$16

5. a. A T-shirt cannot cost –$2.00, although some students may suggest that a store might offer five free T-shirts with $2 rebates to customers who buy such expensive jeans.

b. If J and T are not negative, then their values must fall between two possible extremes: a pair of jeans costs $77 and a T-shirt costs $0, or a T-shirt costs $30.80 and a pair of jeans costs $0.

6. a. $3J + 4T = 182$

b. Answers will vary. Sample responses:

three pairs of jeans for $26 each and four T-shirts for $26 each

three pairs of jeans for $30 each and four T-shirts for $23 each

three pairs of jeans for $34 each and four T-shirts for $20 each

Hints and Comments

Overview

Students explore equations with two unknowns and find values that make the equations true.

About the Mathematics

A combination of values for the unknowns that make an equation true is called a *solution* of the equation. An equation can have many solutions.

Comments About the Solutions

4. This problem provides students with the opportunity to invent their own ways to find one of the many number pairs that make an equation true. They may rely on their invented strategies throughout the unit. Allow students enough time to create their own solutions. Discuss their strategies with the entire class.

One strategy that students may use is the process of fair exchange. Many fair exchanges are possible based on the values of +$5 for the jeans and –$2 for the T-shirts. The context of the problem should make it easy for students to find a fair exchange. A more difficult task is finding one number pair that works so the fair exchange can be applied. Students are given a chance to do this in problem **6**.

5. a. Mathematically speaking, the solution (82, –2) is correct. However, in the context of the problem, the solution does not make sense.

6. b. Here students need to come up with number pairs on their own. Allow them to struggle with the situation.

Note: The term *solution* is introduced on page 3 of the lesson. You may want to discuss this term as students work on these problems.

Solutions and Samples

7. A pair of jeans costs \$42 and a T-shirt costs \$14.

Strategies will vary. Sample strategy:

$3J + 4T = 182$
$2J + 5T = 154$

Exchanging one pair of jeans for one T-shirt decreases the price by \$28. Students can continue this process of exchanging as follows:

$1J + 6T = 126$

$0J + 7T = 98$

The equation, $7T = 98$, can be solved for T ($98 \div 7 = 14$; $T = 14$), and that value can be substituted into either original equation to find J, as follows:

$2J + 5(14) = 154$
$2J + 70 = 154$
$2J = 84$
$J = 42$

8. Explanations will vary. Sample explanation:

First, Hannah wrote Jason's and Antonia's purchases in a table. Next, she created new purchases by tripling Antonia's purchase and doubling Jason's.

On line 5, Hannah found the difference between the new purchases. On line 6, she divided line 5 by seven to find the price of a single T-shirt.

Next, Hannah used the \$14 T-shirt price to find the price of five T-shirts. On the last two lines, she used Antonia's original purchase and the \$70 cost for five T-shirts to find the price of two pairs of jeans. She divided by two to find the price of a single pair of jeans.

9. $2J + 5T = 154$
$3J + 4T = 182$

$$\begin{array}{r} 6J + 15T = 462 \\ \underline{6J + 8T = 364} \\ 0J + 7T = 98 \\ 1T = 14 \end{array}$$

$5T = 70$

$2J + 0T = 84$
$1J = 42$

Hints and Comments

Overview

Students use the information from pages 1–3 of the lesson to find the individual prices for one T-shirt and one pair of jeans, and they review the notebook notation strategy for solving problems.

About the Mathematics

The notebook notation strategy was first introduced in the unit *Comparing Quantities*. It is a way to organize the new combinations that are created by manipulating the known combinations of items. Students can add, subtract, multiply, and divide to find new combinations with the new prices. Eventually they can find a grouping of only one item and then use that to find the price of the individual item.

Comments About the Solutions

7. The strategy described in this problem is the exchange method. Students may use a variety of strategies such as guess-and-check, notebook notation, or a combination chart. The exchange strategy, developed in the unit *Comparing Quantities*, is explained in the solutions column.

8–9. In problems **8** and **9**, students review the notebook notation strategy of solving problems. If students have difficulty, you may want to have an extended discussion of these problems.

8. Mastery of the notebook notation strategy is not important. Allow students to use a strategy that makes sense to them. However, make sure they have some understanding of the notebook notation method.

Extension

You may want to have students work on some of the problems from the unit *Comparing Quantities* that involve notebook notation.

Solutions and Samples

10. a. Explanations will vary. Sample explanation:

Selena created the columns by continually adding to the original purchase. She then chose the two rows from each table that had six pairs of jeans in the purchase. Subtracting the purchases, she found that seven T-shirts cost \$98. She then divided by seven to get the price of one T-shirt.

b. Descriptions will vary. Sample description:

To find the price of jeans, substitute $T = 14$ into one of the two original equations:

$3J + 4(14) = 182$
$3J + 56 = 182$
$3J = 126$
$J = 42$

So, jeans cost \$42.

Hints and Comments

Overview

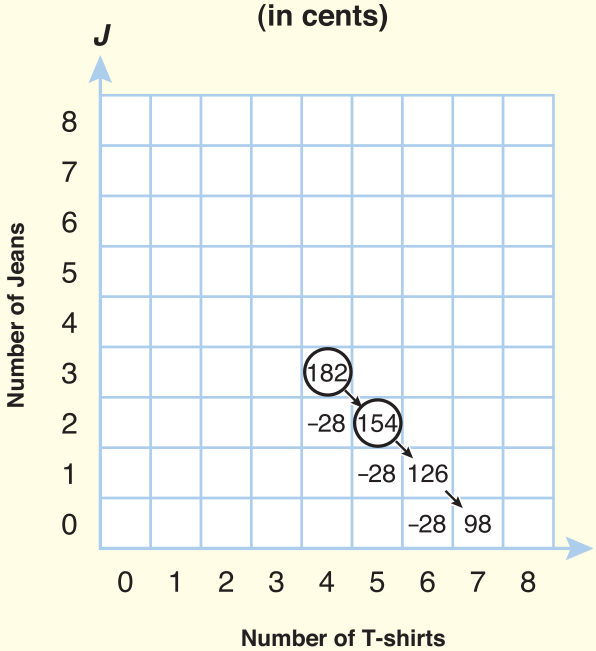
Students fine-tune methods for solving word problems involving two equations with two unknowns.

Comments About the Solutions

10. This problem is critical because students learn a new method for solving a system of two equations with two unknowns. If students have difficulty, you may want to have an extended discussion of this problem.

Some students might connect this doubling process to the process used to construct a ratio table. Other students might use a combination chart such as the following:

Costs of Combinations (in cents)



The original equations are represented by the circled entries. The rows represent numbers of jeans and the columns indicate the numbers of T-shirts. By moving up or down a diagonal, one gets to the left or bottom side of the chart, where there are groups of only one kind of item.

Solutions and Samples

11. A paperback book costs \$3 and an audio book costs \$8. Strategies will vary. Sample strategy:

Using notebook notation:

Books	CDs	Price
2	5	\$46
6	3	\$42
6	15	\$138
6	3	\$42
0	12	\$96
0	1	\$8

Substitute \$8 for the price of one audio book into the equation for the first purchase, as follows:

$2B + 5(8) = 46$
$2B + 40 = 46$
$2B = 6$
$B = 3$
So, a paperback book costs \$3.

12. A watch costs \$18; a ring costs \$11. Strategies will vary. Sample strategy:

Continue from the two given equations by exchanging one watch for one ring, as follows:

$5W + 4R = 134$
$4W + 5R = 127$
$3W + 6R = 120$
$2W + 7R = 113$
$W + 8R = 106$
$0 + 9R = 99$
$R = 11$

If $R = 11$, then
$5W + 4(11) = 134$
$5W = 90$
$W = 18$

13. a. by subtracting equations, as follows:

$$\begin{array}{r} 15J + 20T = 910 \\ 8J + 20T = 616 \\ \hline 7J = 294 \\ J = 42 \end{array}$$

b. substitute the price of a T-shirt into any equation with both J and T, as follows:

$2J + 5T = 154$
$2J + 70 = 154$
$2J = 84$
$J = 42$

Hints and Comments

Overview

Students solve two more shopping problems and use equations to formalize the strategy introduced on page 5 of the lesson.

About the Mathematics

Being able to interpret a situation, translate it to a mathematical model, and solve it is an important skill that students must use to solve problems **11** and **12**. The strategy students used on page 5 of the lesson is now further formalized by using equations. The term *common solution* is defined as "the pair of numbers that makes both equations true."

Comments About the Solutions

11–12. Students can use any strategy to solve these problems. When discussing the problems, you may want to collect different strategies that students use, talk about them, and have students discuss which strategy is most helpful and why.

12. Informal Assessment

This problem assesses students' ability to find a common solution to a pair of equations algebraically, to interpret and organize information presented in a story in mathematical terms, and to model situations using algebra to solve simple linear programming problems.

You may want to ask students which item is likely to be most expensive before they start solving the problem.

13. You may want to use problem **13** in combination with problem **10**, on page 5 of the lesson. If so, discuss the differences and similarities between these two problems.

Extension

You may want to ask students to create their own shopping problem and then solve it in at least two different ways.

Solutions and Samples

14. a. The only common solution to these equations is $X = 15$ and $Y = 40$. Strategies will vary. Sample strategy:

Subtracting equations to solve for Y:

$$\begin{array}{r} X + 2Y = 95 \\ X + Y = 55 \\ \hline Y = 40 \end{array}$$

If $Y = 40$, then

$X + 2(40) = 95$
$X + 80 = 95$
$X = 15$

b. No. Explanations will vary. Sample explanation:

Multiply the first equation by three:

$X + 2Y = 95$
$3X + 6Y = 285$

The difference between that product and the other equation ($3X + 6Y = 290$) shows that no common solution is possible.

c. No. Strategies will vary. Sample strategy:

From problem **14a**, you know that the two equations, $X + 2Y = 95$ and $X + Y = 55$, have a common solution: $X = 15$ and $Y = 40$. Using these values, $3X + Y$ is equal to 85 (45 + 40), not 110.

d. Sandy is incorrect. Explanations will vary. Sample explanation:

In the previous question, if the last equation had been $3X + Y = 85$, then $X = 15$ and $Y = 40$ would have solved all three equations.

15. a. The two statements contain the same information. The second equation is the same as the first equation multiplied by two. One equation is not enough to find the two prices, since there are many pairs of numbers that satisfy the equation.

b. One glass of cranberry juice is $0.80. Strategies will vary. Sample strategy:

Exchange one glass of cranberry juice for one glass of orange juice, while decreasing the price by $0.40.

$2C + 3O = 5.20$
$3C + 2O = 4.80$
$4C + 1O = 4.40$
$5C + 0O = 4.00$
So, $C = \$0.80$

Hints and Comments

Overview

Students solve problems that involve two equations with two unknowns. They look for the common solutions to the two equations.

About the Mathematics

Two equations can have a single common solution when the equations are independent. Being independent means that one equation is not a multiple of the other equation. In problem **15**, the second equation ($4C + 6O = \$10.40$) is double the first ($2C + 3O = \$5.20$), so there is no single common solution.

The graphs of two equations that are not parallel have a single point of intersection. It is possible for three (or more) equations to have one common solution. The graphs of all the equations would intersect at a single point.

Comments About the Solutions

14–15. Encourage students to explain what is going on in each situation. After students have worked on these problems, you may want to discuss dependent and independent equations. Explain that an equation that is a multiple of another equation contains the same information and does not provide any additional information.

14. Because this problem is devoid of context, students must investigate the nature of the equations symbolically, by reasoning about common solutions.

15. b. If students choose to use the table method of solving this problem, they should write the second relationship as $O - C = 0.40$. If students are having difficulty, you may want to encourage them to work directly within the context of the problem.

Solutions and Samples

16. $X = 20$ and $Y = 12$. Strategies will vary. Sample strategy:

$$\begin{array}{ll} (3X + 8Y = 156) \times 2 & 6X + 16Y = 312 \\ (2X + 5Y = 100) \times 3 & \underline{6X + 15Y = 300} \\ & \quad\quad Y = \ \ 12 \end{array}$$

Substituting $Y = 12$ into the first equation, we get the following:

$2X + 5(12) = 100$
$2X + 60 = 100$
$2X = 40$
$X = 20$

For Further Reflection

No. Explanations will vary. Sample explanation:

Two equations could say the same thing, as was the case in the cranberry/orange juice example. Here are the two equations from that example:

$2X + 3Y = 5.20$

$4X + 6Y = 10.40$

These two equations have many common solutions.

Another possibility is that the two equations may say contradictory things. The following is one example:

$2X + 3Y = 10$ and $2X + 3Y = 15$

This situation may represent shopping at two different stores that have different prices.

Hints and Comments

Overview

Students read the Summary, which reviews the mathematical concepts covered in the lesson. Students reflect on different ways to find the common solution of two equations with two unknowns. They explain why not every pair of equations with two unknowns has a single common solution.

About the Mathematics

In this lesson, several strategies for solving two equations with two unknowns have been used. All strategies use the same principle: Make new combinations by combining (adding, subtracting, multiplying, or dividing) combinations (or equations).

Comments About the Solutions

16. Informal Assessment

This problem assesses students' ability to find a common solution to a pair of equations algebraically.

You may want to discuss the strategies students used to solve the problem in this section.

Writing Opportunity

You may have students write their answers to the reflection question in their journals.